ISyE 6663 – Nonlinear Optimization, Spring 2023

Class times, location: MW 2:00–3:15 pm, IC 105 Instructor: Dr. Johannes Milz email: johannes.milz@isye.gatech.edu Office: Groseclose 444 Office Hours: MW 3:30 pm–4:30 pm or by appointment TA: TBD TA email: TBD TA Office Hours and Location: TBD

Course Description

The course provides an introduction to the fundamental theory of convex and nonlinear programming and covers basic algorithms of continuous optimization.

Prerequisites

Linear Algebra (vectors, matrices, linear independence) and basic knowledge of real analysis and multivariate calculus (sequences, subsequences, accumulation/limit points, gradients and Hessian matrices of multivariate functions). The appendix of the course's main reference (see below) provides a comprehensive overview of these topics.

Course Goals and Intended Learning Outcomes

Upon successful completion of the course students will be able to:

- apply fundamental concepts from convex analysis to analyze convexity properties of sets and functions
- understand the role of optimality conditions for convex programming and apply them to solve specific convex optimization problems
- derive first-order and second-order optimality conditions for specific nonlinear optimization problems
- describe first-order and second-order optimization methods for unconstrained optimization in terms of their basic algorithmic design
- understand convergence analyses of optimization methods for unconstrained optimization

Course Content

- Convex sets (basic properties, Helly's theorem, separation theorem, supporting planes)
- Convex functions (basic properties, characterizations of convexity, optimality conditions, subgradients)
- Lagrangian duality (convex theorem on alternative, duality, saddle points, KKT conditions)
- Optimality conditions for nonlinear optimization (first-order and second-order conditions)
- First-order methods for unconstrained minimization (gradient descent, conjugate gradients)
- Second-order methods for unconstrained minimization (Newton's method and its variants)
- Overview of optimization methods for constrained nonlinear optimization (barrier, penalty, augmented Lagrangian, and SQP methods)

Course Requirements & Grading

Grading Policy

The grade will be determined from the following grading scheme:

| homework assignments | 40% |
|----------------------|-----|
| midterm exam | 25% |
| final exam | 35% |

Grading Scale

Your final grade will be assigned as a letter grade according to the following scale:

A 90-100% B 80-89% C 70-79% D 60-69%

F 0-59%

Homework

Homework will be assigned approximately every two weeks and will be due electronically via Canvas as a single PDF by 11:59pm on the due date indicated on each homework. This will typically be Wednesdays. The homework assignments are equally weighted. Each homework problem is graded on a scale of $\{0, 1, 2\}$. After the due date detailed solutions to the homework problems will be made available electronically. You are welcome, even encouraged, to use LATEX to typeset your homework, but handwritten homework is also OK.

Exams

The exams will cover the material discussed in class and in homework problems. Exams will be open notes and open book. The 75 minutes midterm exam is scheduled for March 1, 2023 in class. The final exam is a two hour and fifty minutes exam and is scheduled for April 28, 2023 2:40 PM - 5:30 PM in the class room.

Course Materials

Course Text

There is no official textbook for the course. The course is mainly based on the reference

• A. Ben-Tal and A. Nemirovski: Optimization III: Convex Analysis, Nonlinear Programming Theory, Standard Nonlinear Programming Algorithms, lecture notes, 2023, https://www2.isye.gatech.edu/~nemirovs/

I will provide lecture notes based on this reference. The lecture notes will be made available electronically on Canvas. Homework problems will be taken from the problem sets provided in the lecture notes.

Course Website

I will be using the Course's Canvas Page throughout the semester. On the Canvas page, you should be able to access important information about the course including but not limited to the most updated version of the syllabus and lecture notes, solutions to homeworks as well as course announcements. I will also use the Canvas page to provide feedback on your homework assignments and record grades. Please make sure that you check the Canvas system regularly so you do not miss out on important course information.

Feedback and Office Hours

Office Hours: I am available to meet with you during office hours and after class to answer any questions you may have or to discuss any aspect of the course. If you would like to meet virtually, please send me an email and I will make the necessary arrangements using BlueJeans. You are also welcome to stop by my office anytime before or after the office hours. If my office door is open, the changes are high that I will have time for you.

Email Policy: Email is the best way to contact me outside of class. To ensure that I do not accidentally overlook your email, it would be helpful if you could include the course number in the subject line.

Course Expectations & Guidelines

Academic Integrity

Georgia Tech aims to cultivate a community based on trust, academic integrity, and honor. Students are expected to act according to the highest ethical standards. For information on Georgia Tech's Academic Honor Code, please visit http://www.catalog.gatech.edu/policies/honor-code/ or http://www.catalog.gatech.edu/rules/18/. Any student suspected of cheating or plagiarizing on an exam or assignment will be reported to the Office of Student Integrity, who will investigate the incident and identify the appropriate penalty for violations.

Accommodations for Students with Disabilities

If you are a student with learning needs that require special accommodation, contact the Office of Disability Services at (404)894-2563 or http://disabilityservices.gatech.edu/, as soon as possible, to make an appointment to discuss your special needs and to obtain an accommodations letter. Please also e-mail me as soon as possible in order to set up a time to discuss your learning needs.

Participation

While attendance is not mandatory, I strongly encouraged you to attend the lectures. All students are expected to: (i) complete and submit all homework assignments on time, and (ii) complete the midterm and final exams at their scheduled times.

Collaboration & Group Work

Students may not copy solutions from any source, but are encouraged to use academic references, such as textbooks or research papers as needed. You are encouraged to discuss homework problems with your classmates and learn from each other, but each person must write up and submit their own homework solutions.

Extensions, Late Assignments, & Re-Scheduled/Missed Exams

Late assignments will be accepted only and make-up exams will be given only for documented reasons of illness, family emergency, or participation in approved Institute activities (see http://catalog.gatech.edu/rules/12/ for more information). Final exams will be re-scheduled in accordance with Georgia Tech's policy, also found at http://www.catalog.gatech.edu/rules/12/.

Student-Faculty Expectations Agreement

At Georgia Tech we believe that it is important to strive for an atmosphere of mutual respect, acknowledgement, and responsibility between faculty members and the student body. See http://www.catalog. gatech.edu/rules/22/ for an articulation of some basic expectation that you can have of me and that I have of you. In the end, simple respect for knowledge, hard work, and cordial interactions will help build the environment we seek. Therefore, I encourage you to remain committed to the ideals of Georgia Tech while in this class.

Regrading

Regrade requests will be accepted via email to the instructor within 48 hours of the graded assignment/exam being returned. Requests must include (i) a PDF of the original submission and (ii) a IAT_EX -produced PDF detailing which problems were graded incorrectly and an argument that the submitted solution is indeed correct. Regrades may only be requested if it is believed that a correct answer was marked as incorrect, not because insufficient partial credit was given to an incorrect or partially correct solution. If you request a regrade, you accept that the entire assignment/exam will be regraded, not just the problem(s) believed to be graded incorrectly.

Tentative Course Schedule

Class days are numbered sequentially and printed in **bold**. The course schedule is tentative.

| Monday | TUESDAY | WEDNESDAY | THURSDAY | Friday |
|---|---------|---|----------|--------|
| Jan 9th 1 Introduction and applications | 10th | 11th2HW1 assignedConvex sets:Definition andexamples | 12th | 13th |
| 16th Martin Luther King, Jr. National Holiday | 17th | 18th3Convex sets:Examples, cones,convexity-preservingoperations | 19th | 20th |
| 23rd4Convex sets:closures andinteriors,polyhedral sets,applications | 24th | 25th5HW1 dueHW2 assignedConvex sets:Caratheodory andRadon theorem | 26th | 27th |
| 30th6Convex sets:Helly theorem,applications | 31st | Feb 1st 7 Separation of convex sets | 2nd | 3rd |

| Monday | TUESDAY | WEDNESDAY | THURSDAY | Friday |
|--------------------|---------------|-------------------|--------------|--------------|
| 6th 8 | 7th | 8th 9 | 9th | 10th |
| Separation of | | HW2 due | | |
| convex sets, | | HW3 assigned | | |
| supporting | | Convex functions: | | |
| hyperplanes | | definition, | | |
| | | examples. | | |
| | | convexity- | | |
| | | preserving | | |
| | | operations | | |
| 1941 10 | 1 4+1- | 1541 11 | 1641 | 1741 |
| 13tn 10 | 1401 | 15th 11 | lotn | 17tn |
| Convex functions: | | Convex functions: | | |
| differential | | Minima and | | |
| characterizations | | Optimization | | |
| 20th 12 | 21st | 22nd 13 | 23rd | 24th |
| Convex functions: | | HW3 due | | |
| Optimization and | | Convex theorem | | |
| subgradients | | on alternative | | |
| 0741 14 | 0041 | M 1-4 1F | 0 1 | 91 |
| 27 th 14 | 28th | Mar 1st 15 | 2nd | ara |
| Lagrangian | | Midterm, in | | |
| duality and saddle | | class | | |
| points | | HW4 assigned | | |
| 6th 16 | 7th | 8th 17 | 9th | 10th |
| Saddle points and | | Nonlinear | | |
| optimality | | optimization: | | |
| conditions | | Optimality | | |
| | | conditions | | |
| 13th 18 | 14th | 15th 19 | 16th | 17th |
| Nonlinear | | HW4 due | | |
| optimization | | HW5 assigned | | |
| Optimization. | | Nonlinear | | |
| conditions | | optimization | | |
| conditions | | Optimization. | | |
| | | conditions | | |
| 20+h | 91 <i>a</i> t | 20md | 0.2md | 24+b |
| | | | | |
| Spring Break | Spring Break | Spring Break | Spring Break | Spring Break |
| 27th 20 | 28th | 29th 21 | 30th | 31st |
| Gradient methods | | Gradient methods | | |
| | | | | |
| Apr 3rd 22 | 4th | 5th 23 | 6th | 7th |
| Conjugate | | HW5 due | | |
| gradient method | | HW6 assigned | | |
| Bradient method | | Newton methods | | |
| 10th 24 | 11th | 12th 25 | 13th | 14th |
| Nowton methods | | Oussi Newton | | |
| Trewton methous | | methods | | |
| 1 | 1 | | | 1 |

| Monday | | TUESDAY | Wednesday | THURSDAY | Friday |
|---|----|---------|---|---|--|
| 17th | 26 | 18th | 19th 27 | 20th | 21st |
| Cubic Newton methods | | | HW6 due Overview of methods for constrained nonlinear programming | | |
| 24th Overview of methods for constrained nonlinear programming | 28 | 25th | 26th Reading Period No exams all day | 27th Reading Period No exams 8:00 am to 2:40 pm. | 28th Final exam 2:40 pm – 5:30 pm in IC 105 |